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ANALYSIS OF THE NUMBER OF COMPOSITE NUMBERS IN A GIVEN INTERVAL

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Abstract. In this study, methods were proposed to analyze the number of composite numbers in a given interval and experimental results were carried out.

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1 Introduction

Number theory plays an important role in the fields of Communication and Modern Cryptography (Crandall & Pomerance, 2005). In many Number Theory Problems, it is necessary to state the number of Prime Numbers in a given interval (Dickson, 2005). In such cases, we can use the number of Composite numbers in this interval to calculate the number of Prime numbers in a given interval (Nuri et al., 2019).

Computational schema based on the Dynamic Programming Technique (Bellman & Dreyfus, 1962) to calculate the number of composite numbers in a given interval was proposed in the study Nuri et al. (2019). The proposed method is a continuation of our article in Nuri et al. (2019) and therefore we will benefit from the markings there.

In this study, the number of composite numbers in a given interval is analyzed. For this purpose, the concepts of density and medium density of numbers in a given interval is defined, methods were developed to calculate these parameters, and based on these concepts, the relationship between the numbers of composite numbers in different specific intervals was determined. The results are given by making calculations with proposed methods.

2 Notations

$N = \{1,\ 2,\ .\ .\ .,\ n,\ .\ .\ .\}$, Set of Natural Numbers.

$\overline{N} = N \backslash \{1\} = \{2,\ 3,\ 4,\ \ldots,\ n,\ . ..\}$.

$P = \{p_1,\ p_2,\ p_3,\ \ldots\ \} = \{2,\ 3,\ 5,\ \ldots\}$, Set of Prime Numbers.

$M = \overline{N} \backslash P = CoP$, Set of Composite Numbers.

$M_k = \{m \in \overline{N} |\ m = k \cdot n,\ n \in N\}$, $k \in \overline{N}$;

A set produced by k, set of multiples of k.

$\overline{M_k} = M_k \setminus \{k\}$, $k \in \overline{N}$.

$N_k = \left\{n \in N | k^2 \leq n < (k+1)^2\right\}, k \in N$ A set of Integers from k^2 to $(k+1)^2$.

${T_i}^j = M_i \cap N_j, i \in \overline{N}, \quad j \in N$, Set of integers consisting of multiples of i from k^2 to $(k+1)^2$.

$${U_j}^k = \bigcup_{\substack{p_i \in P \\ i \leq k}} {T_{p_i}}^j, j \in \overline{N};$$

the set of the first k number of p_i's from j^2 to $(j+1)^2$, i. e. the set of composite numbers that are composed of multiples of the first k number of p_i from j^2 to $(j+1)^2$.

$k!! = p_1 \cdot p_2 \cdot p_3 . \cdot ... \cdot p_k$, Prime Factorial - factorial of first k number of primes. Here, p_k is the k.th prime number.

For example,
$8! = p_1 \cdot p_2 \cdot ... \cdot p_8 = 2 \cdot 3 \cdot 5 \cdot 7 \cdot 11 \cdot 13 \cdot 17 \cdot 19$.

$k!!^{(-1)} = (p_1 - 1) \cdot (p_2 - 1) \cdot (p_3 - 1) \cdot \cdots \cdot (p_k - 1)$, multiplication of one less of first k prime numbers. Here, p_k is the k.th prime number.

For example,
$5!!^{(-1)} = (p_1 - 1) \cdot (p_2 - 1) \cdot ... \cdot (p_5 - 1) = (2-1) \cdot (3-1) \cdot (5-1) \cdot (7-1) \cdot (11-1) = 1 \cdot 2 \cdot 4 \cdot 6 \cdot 10$.

3 Some Propositions

Lets's $s(N_k)$ be a number of elememts of a set N_k.

Proposition 1:

$$s(N_k) = 2k + 1, k \in N. \tag{1}$$

Proposition 2: $M = \bigcup_{p \in P} \overline{M_p}$.

Proposition 3: $P = \overline{N} \setminus M = \overline{N} \setminus (\bigcup_{p \in P} \overline{M_p})$ (Sieve of Eratosthenes).

Proposition 4: $P_j = N_j \setminus {U_j}^t$, $j \in \overline{N}$; here t is a number of primes smaller than j, P_j is a set of primes in interval N_j.

4 Density

Let's define a sequence A_k as follows:

$$A_k = \left\{a_i^k \in N |\ a_i^k = k \cdot i, \quad i \in N\right\}, k \in \overline{N}.$$

Let's $A_k(n)$ be a set of numbers of A_k not greater than n.

$$1 \leq a_i^k \leq n, i \in N, k \in \overline{N}.$$

$$0 \leq s\left(A_k\left(n\right)\right) < n \implies 0 \leq \frac{s\left(A_k\left(n\right)\right)}{n} < 1.$$

Let's $d_k(n) = s\left(A_k(n)\right)/n$, $k = 1,\ 2,\ 3, \ldots$.

Let's $d_k(n)$ be *a density of a sequence* A_k *in interval* $[1, n]$.

$$d_k = \mathrm{d(A_k)} = \lim_{n\to\infty}\left\{d_k(n)\right\} = \lim_{n\to\infty}\left\{s\left(A_k(n)\right)/n\right\}, k = 1,\ 2,\ 3, \ldots \tag{2}$$

Let's d_k be the *medium density* of a sequence A_k in the set of N (Natural Numbers).

Proposition 5:

$$d\left(A_k\right) = d_k = \frac{1}{k}\ , k \in \overline{N} \tag{3}$$

Now let's consider the concepts of density of combination of several sequences:
Let's $A^{(k)} = \bigcup_{i=1}^{k} A_{p_i}$.

For example,
for $k = 1$, $A^{(1)} = A_{p_1} = A_2$,
for $k = 2$, $A^{(2)} = A_{p_1} \cup A_{p_2} = A_2 \cup A_3$,
for $k = 3$, $A^{(3)} = A_{p_1} \cup A_{p_2} \cup A_{p_3} = A_2 \cup A_3 \cup A_5$,
for $k = 4$, $A^{(4)} = A_{p_1} \cup A_{p_2} \cup A_{p_3} \cup A_{p_4} = A_2 \cup A_3 \cup A_5 \cup A_7$,
for $k = 5$, $A^{(5)} = A_{p_1} \cup A_{p_2} \cup A_{p_3} \cup A_{p_4} \cup A_{p_5} = A_2 \cup A_3 \cup A_5 \cup A_7 \cup A_{11}$ and etc.

Let's $A^{(k)}(n)$ be a set of elements not greater than n of set $A^{(k)}$.
Density of $A^{(k)}$ *in an interval* $[1, n]$ is

$$d^{(k)}(n) = d\left(A^{(k)}(n)\right) = s\left(A^{(k)}(n)\right)\Big/n,$$

medium density is

$$d^{(k)} = d\left(A^{(k)}\right) = \lim_{n\to\infty}\left\{d^k(n)\right\} = \lim_{n\to\infty}\left\{s\left(A^{(k)}(n)\right)/n\right\} \tag{4}$$

5 Recursive Method for Calculating Medium Density $A^{(k)}$

As it is known (Rosen, 2012) the number of elements of the combinations of two sets and n sets is calculated with the formulas (5) and (6) given below:

$$s\left(A \cup B\right) = s\left(A\right) + s\left(B\right) - s\left(A \cap B\right) \tag{5}$$

$$\begin{aligned} s\left(\cup_{i=1}^{n} A_i\right) = & \sum_{i=1}^{n} s(A_i) - \sum_{i=1}^{n-1}\sum_{j=i+1}^{n} s(A_i \cap A_j) \\ & + \sum_{i=1}^{n-2}\sum_{j=i+1}^{n-1}\sum_{k=j+1}^{n} s(A_i \cap A_j \cap A_k) \\ & - \sum_{i=1}^{n-3}\sum_{j=i+1}^{n-2}\sum_{k=j+1}^{n-1}\sum_{l=k+1}^{n} s(A_i \cap A_j \cap A_k \cap A_l) \\ & + \sum_{i=1}^{n-4}\sum_{j=i+1}^{n-3}\sum_{k=j+1}^{n-2}\sum_{l=k+1}^{n-1}\sum_{t=l+1}^{n} s(A_i \cap A_j \cap A_k \cap A_l \cap A_t) \\ & + \cdots + (-1)^{n+1} \cdot s(A_1 \cap A_2 \cap A_3 \cap \cdots \cap A_{n-1} \cap A_n) \end{aligned} \tag{6}$$

According to the formulas given in (4) and (5) we can write the following:

$$d(A \cup B) = d(A) + d(B) - d(A) \cdot d(B) \tag{7}$$

$$\begin{aligned} d(A_k \cup A_s) &= d(A_k) + d(A_s) - d(A_k \cap A_s) = \\ &= d(A_k) + d(A_s) - d(A_k) \cdot d(A_s) = \\ &= \ d_k + d_s - d_k \cdot d_s = d_k + d_s \cdot (1 - d_k) \end{aligned} \tag{8}$$

Let's calculate the following parameters based on the formula (8):

Because, $A^{(1)} = A_1 \implies d^{(1)} = d\left(A^{(1)}\right) = d(A_1) = d_1 = 1/p_1 = 1/2$.

$$d^{(2)} = d\left(A^{(2)}\right) = d(A_{p_1} \cup A_{p_2}) = d(A_2 \cup A_3) = d(A_2) + d(A_3) - d(A_2) \cdot d(A_3) =$$

$$= d_2 + d_3 \cdot (1 - d_2) = d^{(1)} + \frac{1}{p_2} \cdot \left(1 - d^{(1)}\right) = \frac{1}{2} + \frac{1}{3} \cdot \left(1 - \frac{1}{2}\right) = \frac{4}{6}$$

$$d^{(3)} = d\left(A^{(3)}\right) = d(A_{p_1} \cup A_{p_2} \cup A_{p_2}) = d(A_2 \cup A_3 \cup A_5) = d^{(2)} + \frac{1}{p_3} \cdot \left(1 - d^{(2)}\right) =$$

$$= \frac{4}{6} + \frac{1}{5} \cdot \left(1 - \frac{4}{6}\right) = \frac{22}{30}$$

$$d^{(4)} = d\left(A^{(4)}\right) = d(A_{p_1} \cup A_{p_2} \cup A_{p_2} \cup A_{p_3}) = d(A_2 \cup A_3 \cup A_5 \cup A_7) =$$

$$= d^{(3)} + \frac{1}{p_7} \cdot \left(1 - d^{(3)}\right) = \frac{22}{30} + \frac{1}{7} \cdot \left(1 - \frac{22}{30}\right) = \frac{162}{210}$$

By generalizing the formula (8) lets define general recursive formula (13) to calculate the medium density for any n.

$$d\left(A^1\right) = 1/p_1 = 1/2 = 1/p_1 = c_1/e_1$$

$$c_1 = 1, e_1 = 2$$

$$d\left(A^{k+1}\right) = c_{k+1}/e_{k+1}, \quad k \in N \tag{9}$$

$$c_{k+1} = c_k \cdot (p_{k+1} - 1) + e_k \tag{10}$$

$$e_k = p_1 \cdot p_2 \cdot \ldots \cdot p_k = k!! \tag{11}$$

$$c_{k+1} = c_k \cdot (p_{k+1} - 1) + p_1 \cdot p_2 \cdot \ldots \cdot p_k = c_k \cdot (p_{k+1} - 1) + k!!$$

$$d\left(A^{k+1}\right) = (c_k \cdot (p_{k+1} - 1) + p_1 \cdot p_2 \cdot \ldots \cdot p_k)/(p_1 \cdot p_2 \cdot \ldots \cdot p_k . p_{k+1}), k \in N \tag{12}$$

$$d\left(A^{k+1}\right) = (c_k \cdot (p_{k+1} - 1) + k!!)/((k+1)!!), k \in N \tag{13}$$

The accuracy of the formulas (10) and (12) derived from the equation (14) shown below:

$$\begin{aligned} d^{(k+1)} = d^{(k)} + \frac{1}{p_{k+1}} \cdot \left(1 - d^{(k)}\right) = \frac{c_k}{e_k} + \frac{1}{p_{k+1}} \cdot \left(1 - \frac{c_k}{e_k}\right) &= \frac{c_k \cdot p_{k+1} + (e_k - c_k)}{e_k \cdot p_{k+1}} = \\ &= \frac{c_k \cdot (p_{k+1} - 1) + e_k}{e_{k+1}} \end{aligned} \tag{14}$$

6 Direct Method to Calculate Medium Density of A^k

An easier method for calculating the medium density is proposed below:
We can write the formula (7) as follows:

$$1 - d\left(A \cup B\right) = \left(1 - d\left(A\right)\right) \cdot \left(1 - d\left(B\right)\right)$$

If we generalize this expression for k number of sets, we get the following formula:

$$1 - d\left(A_1 \cup A_2 \cup \ldots \cup A_k\right) = \prod_{i=1}^{k} \left(1 - d\left(A_i\right)\right)$$

$$d\left(A_1 \cup A_2 \cup \ldots \cup A_k\right) = 1 - \prod_{i=1}^{k} \left(1 - d\left(A_i\right)\right)$$

$A^{(1)} = A_1$, $d\left(A^{(1)}\right) = d\left(A_1\right) = d_1 = 1/p_1 = 1/2$. Let's $r_1 = \left(p_1 - 1\right)/p_1 = \left(2 - 1\right)/2 = 1/2$. Then $d\left(A^{(1)}\right) = 1 - r_1 = 1 - 1/2 = 1/2$.

If we generalize this expression for k number of sets, we get the following formulas (15) and (16) to calculate the medium density:

$$d\left(A^{(k)}\right) = 1 - r_k, k = 1,\ 2,\ 3,\ \ldots \tag{15}$$

$$r_k = r_{k-1} \cdot \left(\left(p_k - 1\right)/p_k\right), k = 1,\ 2,\ 3, \ldots$$

$$r_k = \left(\left(p_1 - 1\right) \cdot \left(p_2 - 1\right) \cdot \ldots \cdot \left(p_k - 1\right)\right) / \left(p_1 \cdot p_2 \cdot \ldots \cdot p_k\right)$$

$$r_k = \left(\prod_{i=1}^{k} \left(p_i - 1\right)\right) \Big/ \left(\prod_{i=1}^{k} p_i\right) = (k!!^{(-1)})/(k!!) \tag{16}$$

As it is seen, formulas (15) and (16) also has a recursive feature, but unlike formula (10) (13), formulas (15) and (16) can be calculated for each k value directly, that is, without knowing the previous $(k-1)$ number of r_k values.

Let's calculate $d\left(A^{(k)}\right), (k = 2, 3, 4)$ with the formulas (15) and (16).

$d\left(A^{(2)}\right) = 1 - r_2, r_2 = r_1 \cdot \left(\left(p_2 - 1\right)/p_2\right) = 1/2 \cdot \left(\left(3 - 1\right)/3\right) = 1/2 \cdot 2/3 = 2/6$,
$d^{(2)} = 1 - 2/6 = 4/6$.

$d\left(A^{(3)}\right) = 1 - r_3, r_3 = r_2 \cdot \left(\left(p_3 - 1\right)/p_3\right) = 4/6 \cdot \left(\left(5 - 1\right)/5\right) = 2/6 \cdot 4/5 = 8/30$,
$d^{(3)} = 1 - 8/30 = 22/30$.

$d\left(A^{(4)}\right) = 1 - r_4, r_4 = r_3 \cdot \left(\left(p_4 - 1\right)/p_4\right) = 8/30 \cdot \left(\left(7 - 1\right)/7\right) = 8/30 \cdot 6/7 = 48/210$,
$d^{(4)} = 1 - 48/210 = 162/210$.

7 Analysis of the Relationship Between s($U_j{}^k$)'s

Let's $u_j^k = s\left(U_j^k\right)$. The following theorem is true:
Theorem 6: $u_{j+k!!}^k = u_j^k + 2 \cdot c_k$, $k \in N$, $j \in N$.

For values $k = 1, 2, \ldots, 8$ theorem will be as following:
for $k = 1$, $u_{j+2}^1 = u_j^1 + 2, \quad j \in N$

for $k = 2$, $u^2_{j+6} = u^2_j + 8, \quad j \in N$
for $k = 3$, $u^3_{j+30} = u^3_j + 44, \quad j \in N$
for $k = 4$, $u^4_{j+210} = u^4_j + 324, \quad j \in N$
for $k = 5$, $u^5_{j+2310} = u^5_j + 3660, \quad j \in N$
for $k = 6$, $u^6_{j+30030} = u^6_j + 48540, \quad j \in N$
for $k = 7$, $u^7_{j+510510} = u^7_j + 836700, \quad j \in N$
for $k = 8$, $u^8_{j+9699690} = u^8_j + 16081620, \quad j \in N$

Proof: According to definitions, ${U_j}^k = \bigcup_{\substack{p_i \in P \\ i \leq k}} {T_{p_i}}^j,\ j \in \overline{N}$;

Here, ${T_{p_i}}^j = M_{p_i} \cap N_j,\ i \in N, \quad j \in N$;

$$N_j = \left\{ n \in N | j^2 \leq n < (j+1)^2 \right\}, j \in N;$$

$$M_{p_i} = \left\{ m \in \overline{N} |\ m = p_i \cdot n,\ n \in N \right\}, i \in N;$$

Here, $M_{p_i} = A_{p_i}$, $i \in N$ and $U^k_j = A^k \cap N_j$, $U^k_{j+k!!} = A^k \cap N_{j+k!!}$, $k \in N$, $j \in N$.

According to proposition 1, $s\left(N_j\right) = 2j + 1$, $j \in N$ and

$$s(N_{j+k!!}) = 2(j + k!!) + 1 = (2j + 1) + 2k!!, \quad j \in N \tag{17}$$

According to equations (9) and (11), $d\left(A^k\right) = c_k/e_k$, $k \in N$ ve $e_k = p_1 \cdot p_2 \cdot \ldots \cdot p_k = k!!$

$$d\left(A^k\right) = c_k/k!! \tag{18}$$

According to equations (17) and (18):

$$u^k_{j+k!!} = s\left(U^k_{j+k!!}\right) = s\left(U^k_j\right) + d\left(A^k\right) \cdot (2k!!) = s\left(U^k_j\right) + (c_k/k!!) \cdot (2k!!) = u^k_j + 2 \cdot c_k$$

8 Computational Results

To give the results of Theorem 6 visually, the values of $s(U^k_j)$, $k \in N$, $j \in N$ were calculated using the NNN algorithm proposed in the study Nuri et al. (2019) and some part of the results are given below in Table 1 - Table 6. All tables are given for the values $k = 1, 2, 3, 4$, the index j changes from 1 - 45 in Table 1, 46 - 90 in Table 2, 91 - 135 in Table 3, 136 - 180 in Table 4, 181 - 225 in Table 5, 226 - 270 in Table 6.

As you can see from the tables, the values increase 2 by every 2 rows in the first column.

$$u^1_{j+2} = u^1_j + 2,\ j = 1,\ 2,\ \ldots,\ 268$$

In other words, the values in the $1., 3., 5.,$ ....th rows for u^1_j increase as 1, 3, 5 ... and the values in 2., 4., 6., ...th rows for u^1_j increase as $3, 5, 7$... and etc.

In the second column, values of u^2_j increase 8 by 6 rows,

$$u^2_{j+6} = u^2_j + 8,\ j = 1,\ 2,\ \ldots,\ 264$$

Table 1: u_j^k $(k = \overline{(1,4)}; j = \overline{(1,45)})$

j	Interval	u_j^1	u_j^2	u_j^3	u_j^4
1	1–3	1	2	2	2
2	4–8	3	3	4	5
3	9–15	3	5	5	5
4	16-24	5	6	6	6
5	25-35	5	7	9	9
6	36-48	7	9	9	9
7	49-63	7	10	11	12
8	64-80	9	11	12	13
9	81-99	9	13	15	16
10	100-120	11	14	15	16
11	121-143	11	15	16	17
12	144-168	13	17	19	20
13	169-195	13	18	20	20
14	196-224	15	19	21	23
15	225-255	15	21	23	23
16	256-288	17	22	24	26
17	289-323	17	23	25	26
18	324-360	19	25	28	30
19	361-399	19	26	29	30
20	400-440	21	27	29	31
21	441-483	21	29	32	33
22	484-528	23	30	33	35
23	529-575	23	31	35	37
24	576-624	25	33	35	37
25	625-675	25	34	38	39
26	676-728	27	35	39	42
27	729-783	27	37	40	42
28	784-840	29	38	42	44
29	841-899	29	39	43	45
30	900-960	31	41	45	48
31	961-1023	31	42	46	48
32	1024-1088	33	43	48	50
33	1089-1155	33	45	49	52
34	1156-1224	35	46	50	53
35	1225-1295	35	47	53	55
36	1296-1368	37	49	53	56
37	1369-1443	37	50	55	58
38	1444-1520	39	51	56	59
39	1521-1599	39	53	59	62
40	1600-1680	41	54	59	62
41	1681-1763	41	55	60	63
42	1764-1848	43	57	63	67
43	1849-1935	43	58	64	66
44	1936-2024	45	59	65	70
45	2025-2115	45	61	67	70

Table 2: u_j^k $(k = \overline{(1,4)}; j = \overline{(46,90)})$

j	Interval	u_j^1	u_j^2	u_j^3	u_j^4
46	2116-2208	47	62	68	71
47	2209-2303	47	63	69	73
48	2304-2400	49	65	72	75
49	2401-2499	49	66	73	77
50	2500-2600	51	67	73	77
51	2601-2703	51	69	76	80
52	2704-2808	53	70	77	81
53	2809-2915	53	71	79	83
54	2916-3024	55	73	79	83
55	3025-3135	55	74	82	86
56	3136-3248	57	75	83	88
57	3249-3363	57	77	84	88
58	3364-3480	59	78	86	91
59	3481-3599	59	79	87	91
60	3600-3720	61	81	89	94
61	3721-3843	61	82	90	94
62	3844-3968	63	83	92	97
63	3969-4095	63	85	93	98
64	4096-4224	65	86	94	99
65	4225-4355	65	87	97	102
66	4356-4488	67	89	97	102
67	4489-4623	67	90	99	104
68	4624-4760	69	91	100	106
69	4761-4899	69	93	103	107
70	4900-5040	71	94	103	109
71	5041-5183	71	95	104	110
72	5184-5328	73	97	107	112
73	5329-5475	73	98	108	114
74	5476-5624	75	99	109	115
75	5625-5775	75	101	111	116
76	5776-5928	77	102	112	117
77	5929-6083	77	103	113	120
78	6084-6240	79	105	116	122
79	6241-6399	79	106	117	123
80	6400-6560	81	107	117	123
81	6561-6723	81	109	120	126
82	6724-6888	83	110	121	128
83	6889-7055	83	111	123	129
84	7056-7224	85	113	123	129
85	7225-7395	85	114	126	132
86	7396-7568	87	115	127	135
87	7569-7743	87	117	128	134
88	7744-7920	89	118	130	137
89	7921-8099	89	119	131	138
90	8100-8280	91	121	133	139

Table 3: u_j^k $(k = \overline{(1,4)}; j = \overline{(91,135)})$

j	Interval	u_j^1	u_j^2	u_j^3	u_j^4
91	8281-8463	91	122	134	141
92	8464-8648	93	123	136	143
93	8649-8835	93	125	137	145
94	8836-9024	95	126	138	145
95	9025-9215	95	127	141	148
96	9216-9408	97	129	141	149
97	9409-9603	97	130	143	150
98	9604-9800	99	131	144	152
99	9801-9999	99	133	147	154
100	10000-10200	101	134	147	155
101	10201-10403	101	135	148	155
102	10404-10608	103	137	151	159
103	10609-10815	103	138	152	160
104	10816-11024	105	139	153	161
105	11025-11235	105	141	155	163
106	11236-11448	107	142	156	164
107	11449-11663	107	143	157	165
108	11664-11880	109	145	160	169
109	11881-12099	109	146	161	169
110	12100-12320	111	147	161	170
111	12321-12543	111	149	164	172
112	12544-12768	113	150	165	174
113	12769-12995	113	151	167	175
114	12996-13224	115	153	167	176
115	13225-13455	115	154	170	179
116	13456-13688	117	155	171	179
117	13689-13923	117	157	172	181
118	13924-14160	119	158	174	183
119	14161-14399	119	159	175	185
120	14400-14640	121	161	177	186
121	14641-14883	121	162	178	187
122	14884-15128	123	163	180	190
123	15129-15375	123	165	181	189
124	15376-15624	125	166	182	192
125	15625-15875	125	167	185	195
126	15876-16128	127	169	185	195
127	16129-16383	127	170	187	196
128	16384-16640	129	171	188	198
129	16641-16899	129	173	191	201
130	16900-17160	131	174	191	201
131	17161-17423	131	175	192	202
132	17424-17688	133	177	195	204
133	17689-17955	133	178	196	207
134	17956-18224	135	179	197	208
135	18225-18495	135	181	199	209

Table 4: u_j^k $(k = \overline{(1,4)}; j = \overline{(136,180)})$

j	Interval	u_j^1	u_j^2	u_j^3	u_j^4
136	18496-18768	137	182	200	210
137	18769-19043	137	183	201	212
138	19044-19320	139	185	204	214
139	19321-19599	139	186	205	215
140	19600-19880	141	187	205	217
141	19881-20163	141	189	208	218
142	20164-20448	143	190	209	220
143	20449-20735	143	191	211	222
144	20736-21024	145	193	211	222
145	21025-21315	145	194	214	225
146	21316-21608	147	195	215	226
147	21609-21903	147	197	216	227
148	21904-22200	149	198	218	230
149	22201-22499	149	199	219	230
150	22500-22800	151	201	221	233
151	22801-23103	151	202	222	233
152	23104-23408	153	203	224	236
153	23409-23715	153	205	225	236
154	23716-24024	155	206	226	238
155	24025-24335	155	207	229	241
156	24336-24648	157	209	229	241
157	24649-24963	157	210	231	243
158	24964-25280	159	211	232	244
159	25281-25599	159	213	235	247
160	25600-25920	161	214	235	247
161	25921-26243	161	215	236	249
162	26244-26568	163	217	239	251
163	26569-26895	163	218	240	253
164	26896-27224	165	219	241	254
165	27225-27555	165	221	243	254
166	27556-27888	167	222	244	258
167	27889-28223	167	223	245	257
168	28224-28560	169	225	248	261
169	28561-28899	169	226	249	262
170	28900-29240	171	227	249	262
171	29241-29583	171	229	252	265
172	29584-29928	173	230	253	266
173	29929-30275	173	231	255	268
174	30276-30624	175	233	255	269
175	30625-30975	175	234	258	271
176	30976-31328	177	235	259	272
177	31329-31683	177	237	260	274
178	31684-32040	179	238	262	276
179	32041-32399	179	239	263	276
180	32400-32760	181	241	265	279

Table 5: u_j^k $(k = \overline{(1,4)}; j = \overline{(91,225)})$

j	Interval	u_j^1	u_j^2	u_j^3	u_j^4
181	32761-33123	181	242	266	280
182	33124-33488	183	243	268	282
183	33489-33855	183	245	269	282
184	33856-34224	185	246	270	285
185	34225-34595	185	247	273	287
186	34596-34968	187	249	273	287
187	34969-35343	187	250	275	289
188	35344-35720	189	251	276	291
189	35721-36099	189	253	279	293
190	36100-36480	191	254	279	294
191	36481-36863	191	255	280	294
192	36864-37248	193	257	283	298
193	37249-37635	193	258	284	298
194	37636-38024	195	259	285	301
195	38025-38415	195	261	287	301
196	38416-38808	197	262	288	304
197	38809-39203	197	263	289	304
198	39204-39600	199	265	292	307
199	39601-39999	199	266	293	308
200	40000-40400	201	267	293	308
201	40401-40803	201	269	296	311
202	40804-41208	203	270	297	312
203	41209-41615	203	271	299	315
204	41616-42024	205	273	299	315
205	42025-42435	205	274	302	318
206	42436-42848	207	275	303	319
207	42849-43263	207	277	304	319
208	43264-43680	209	278	306	322
209	43681-44099	209	279	307	323
210	44100-44520	211	281	309	325
211	44521-44943	211	282	310	326
212	44944-45368	213	283	312	329
213	45369-45795	213	285	313	329
214	45796-46224	215	286	314	330
215	46225-46655	215	287	317	333
216	46656-47088	217	289	317	333
217	47089-47523	217	290	319	336
218	47524-47960	219	291	320	337
219	47961-48399	219	293	323	340
220	48400-48840	221	294	323	340
221	48841-49283	221	295	324	341
222	49284-49728	223	297	327	344
223	49729-50175	223	298	328	344
224	50176-50624	225	299	329	347
225	50625-51075	225	301	331	347

Table 6: u_j^k $(k = \overline{(1,4)}; j = \overline{(226,270)})$

j	Interval	u_j^1	u_j^2	u_j^3	u_j^4
226	51076-51528	227	302	332	350
227	51529-51983	227	303	333	350
228	51984-52440	229	305	336	354
229	52441-52899	229	306	337	354
230	52900-53360	231	307	337	355
231	53361-53823	231	309	340	357
232	53824-54288	233	310	341	359
233	54289-54755	233	311	343	361
234	54756-55224	235	313	343	361
235	55225-55695	235	314	346	363
236	55696-56168	237	315	347	366
237	56169-56643	237	317	348	366
238	56644-57120	239	318	350	368
239	57121-57599	239	319	351	369
240	57600-58080	241	321	353	372
241	58081-58563	241	322	354	372
242	58564-59048	243	323	356	374
243	59049-59535	243	325	357	376
244	59536-60024	245	326	358	377
245	60025-60515	245	327	361	379
246	60516-61008	247	329	361	380
247	61009-61503	247	330	363	382
248	61504-62000	249	331	364	383
249	62001-62499	249	333	367	386
250	62500-63000	251	334	367	386
251	63001-63503	251	335	368	387
252	63504-64008	253	337	371	391
253	64009-64515	253	338	372	390
254	64516-65024	255	339	373	394
255	65025-65535	255	341	375	394
256	65536-66048	257	342	376	395
257	66049-66563	257	343	377	397
258	66564-67080	259	345	380	399
259	67081-67599	259	346	381	401
260	67600-68120	261	347	381	401
261	68121-68643	261	349	384	404
262	68644-69168	263	350	385	405
263	69169-69695	263	351	387	407
264	69696-70224	265	353	387	407
265	70225-70755	265	354	390	410
266	70756-71288	267	355	391	412
267	71289-71823	267	357	392	412
268	71824-72360	269	358	394	415
269	72361-72899	269	359	395	415
270	72900-73440	271	361	397	418

The values in the $1., 7., 13., \ldots.$th rows for u_j^2 increase as $2, 10, 18, \ldots$, values in the $2., 8., 14., \ldots.$th rows for u_j^2 increase as $3, 11, 19, \ldots$, values in the $3., 9., 15., \ldots.$th rows for u_j^2 increase as $5, 13, 21, \ldots$, values in the $4., 10., 16., \ldots.$th rows for u_j^2 increase as $6, 14, 22, \ldots$, values in the $5., 11., 17., \ldots.$th rows for u_j^2 increase as $7, 15, 23, \ldots$, values in the $6., 12., 18., \ldots.$th rows for u_j^2 increase as $9, 17, 25, \ldots.$ and etc.

In the third column values of u_j^3 increse 44 by 30 rows.

$$u_{j+30}^3 = u_j^3 + 44,\ j = 1,\ 2,\ \ldots,\ 240$$

In other words, values in the $1., 31., 61., \ldots.$th rows for u_j^3 increase as $2, 46, 90, \ldots$, values in the $2., 32., 62., \ldots.$th rows for u_j^3 increase as $4, 48, 92, \ldots$, and values in the $3., 33., 63., \ldots.$th rows, values in the $4., 34., 64., \ldots.$th rows and values in the $30., 60., 90., \ldots, 450.$th rows for u_j^3 increase 44.

In the fourth column values of u_j^4 increse 324 by 210 rows.

$$u_{j+210}^4 = u_j^4 + 324,\ j = 1,\ 2,\ \ldots,\ 60$$

In other words, values in the $1., 211.$th rows for u_j^4 increase as $2, 326$, values in the $2., 212.$th rows for u_j^4 increase as $5., 329$ and by this way values in the $3., 213.$th rows, values in the $4., 214., \ldots.$th rows and finally values in the $60., 270.$th rows for u_j^4 increase 324.

9 Conclusion

The results of this study will be used in our subsequent studies to determine the upper bound of composite numbers in a given interval.

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